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Confinement from a scalar-gluon coupling in gauge theory

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Abstract. A model is introduced with a massive scalar coupling to the Yang-Mills term in four-dimensional gauge theory. It is shown that the resulting potential of colour sources consists of a short distance Coulomb interaction and a confining part dominating at large distances. Far away from the source the scalar vanishes $\sim r^{-1}$ while the potential diverges linearly. Up to an N_c-dependent factor of order 1 the tension parameter in the model is gmf, where m denotes the mass of the scalar and f is a coupling scale entering the scalar-gluon coupling.

1. Recently it was observed that a string inspired coupling of a massless dilaton to gauge fields yields a linearly increasing vector potential from pointlike colour sources if a logarithmic divergence of the dilaton at infinity is permitted [1,2]. While the string inspired model also has the virtue to yield a regularization of the Coulomb potential if a solution is sought with the dilaton vanishing at infinity, it can not be solved in the presence of a dilaton mass term, which would define a more realistic and phenomenologically more interesting model. This motivated me to construct a direct coupling of a *massive scalar* to chromo-electric and magnetic fields subject to the requirement that the Coulomb problem still admits an analytic solution. The model for the scalar-gluon coupling that emerged from these efforts is

$$\mathcal{L} = -\frac{1}{4}\frac{\phi^2}{f^2+\beta\phi^2}F_{\mu\nu}{}^{j}F^{\mu\nu}{}_{j} - \frac{1}{2}\partial_\mu\phi\cdot\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \ , \quad (1)$$

where β is a parameter and f is a mass scale characterizing the strength of the scalar-gluon coupling. This new model differs from the previous model in several respects: While in the string inspired model with a massless dilaton the tension parameter turned out to be proportional to the square f^2 of the dilaton coupling scale and is independent of the QCD coupling g, in the new model the tension is proportional to gmf. Moreover, the quark interaction potential in the present model turns out to resemble a Cornell or Buchmüller-Tye type potential [3], which is a phenomenologically attractive property.

To analyze the Coulomb problem in the theory described by (1) we consider a pointlike colour source, which in its rest frame is described by a current

$$j^\mu{}_i = g\delta(\mathbf{r})C_i\eta^\mu{}_0 = \varrho_i\eta^\mu{}_0 \ . \quad (2)$$

Here $1 \le i \le N_c^2 - 1$ is an su(N_c) Lie algebra index and $C_i = \zeta^+ \cdot X_i \cdot \zeta$ is the expectation value of the su(N_c) generator X_i for a normalized spinor ζ in colour space. These expectation values satisfy

$$\sum_{i=1}^{N_c^2-1} C_i^2 = \frac{N_c - 1}{2N_c} \ ,$$

as a consequence of the identity

$$\sum_{i=1}^{N_c^2-1} X_i{}^a{}_b X^{ic}{}_d = \frac{1}{2}\delta^a{}_d\delta^c{}_b - \frac{1}{2N_c}\delta^a{}_b\delta^c{}_d \quad (3)$$

for su(N_c) generators in an N_c-dimensional representation.

The equations of motion for the scalar and gluons emerging from this source follow like in [1]: The equations are for general sources

$$\partial_\mu\Big(\frac{\phi^2}{f^2+\beta\phi^2}F^{\mu\nu}{}_i\Big) + g\frac{\phi^2}{f^2+\beta\phi^2}A_\mu{}^j f_{ij}{}^k F^{\mu\nu}{}_k = -j^\nu{}_i,$$

$$\partial^2\phi = m^2\phi + \frac{f^2\phi}{2(f^2+\beta\phi^2)^2}F_{\mu\nu}{}^j F^{\mu\nu}{}_j,$$

and to explore the impact of the scalar-gluon coupling on the Coulomb potential we consider the Gauss law and Faraday's law for stationary configurations in the presence of the source (2):

$$\nabla\cdot\Big(\frac{\phi^2}{f^2+\beta\phi^2}\mathbf{E}\Big) - ig\frac{\phi^2}{f^2+\beta\phi^2}(\mathbf{A}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{A}) = \varrho \ ,$$
$$\nabla\times\mathbf{E} - ig(\mathbf{A}\times\mathbf{E} + \mathbf{E}\times\mathbf{A}) = 0,$$

where $\varrho = \varrho^i X_i$ and[1] $\mathbf{E} = -F_{0i}{}^j\mathbf{e}^i X_j$.

[1] Note that here i is a spatial index and j is a Lie algebra index. In all other equations in the paper i, j, k always denote Lie algebra indices

The vector potential $\mathbf{A}$ is pure gauge in the static limit since the chromo-magnetic field

$$\mathbf{B} = \nabla \times \mathbf{A} - ig\mathbf{A} \times \mathbf{A}$$

vanishes. Therefore we end up with

$$\nabla \cdot \left(\frac{\phi^2}{f^2 + \beta\phi^2} \mathbf{E}_i \right) = gC_i\delta(\mathbf{r}), \tag{4}$$

$$\Delta\phi = m^2\phi - \frac{f^2\phi}{(f^2 + \beta\phi^2)^2} \mathbf{E}^i \cdot \mathbf{E}_i \ , \tag{5}$$

while Faraday's law reduces to $\nabla \times \mathbf{E}_i = 0$ complying with chromo-electric potentials $\mathbf{E}_i = -\nabla\Phi_i$.

Clearly, the solution to the Coulomb problem (4,5) includes the case of inertial motion of the colour source through appropriate Lorentz boosts.

Equation (4) or more generally its analog for an arbitrary spherically symmetric colour density yields for the fields outside the density

$$\frac{\phi^2}{f^2 + \beta\phi^2} \mathbf{E}_i = \frac{gC_i}{4\pi r^2} \mathbf{e}_r \ ,$$

and inserting this relation in (5) yields

$$\Delta\phi = m^2\phi - \frac{\mu^2}{r^4\phi^3}, \tag{6}$$

where the abbreviation

$$\mu = \frac{gf}{4\pi}\sqrt{\frac{N_c - 1}{2N_c}}$$

was used.

Substituting $y(r) = r\phi(r)$ in (6) and multiplying by dy/dr yields the first integral

$$\left(\frac{dy}{dr}\right)^2 = m^2y^2 + \frac{\mu^2}{y^2} + 2K \ .$$

This can readily be solved for arbitrary integration constant K, but the boundary condition $\lim_{r\to\infty} \phi(r) = 0$ uniquely determines $K = -m\mu$. This yields

$$y^2(r) = \frac{\mu}{m} + \left(y_0^2 - \frac{\mu}{m}\right) \exp(-2mr) \ .$$

Therefore, the scalar field emerging from the pointlike colour source is

$$\phi = \pm\frac{1}{r}\sqrt{\frac{\mu}{m} + \left(y_0^2 - \frac{\mu}{m}\right) \exp(-2mr)} \ , \tag{7}$$

while the chromo-electric potentials consist of a Coulomb and a confining part:

$$\Phi_i = \beta\frac{gC_i}{4\pi r} - fC_i\sqrt{\frac{N_c}{2(N_c - 1)}} \times \ln\left(\exp(2mr) - 1 + \frac{m}{\mu}y_0^2\right) \ . \tag{8}$$

At large distance the scalar field vanishes $\sim r^{-1}$ for $y_0^2 > 0$, while the chromo-electric potentials yield linear confinement in the non-relativistic regime, and also in the relativistic regime if applied in the framework of a reduced Salpeter or no-pair equation [4].

2. Equation (8) implies that a classical (anti-)quark with colour orientation ζ_q, $\zeta_q^+ \cdot \zeta_q = 1$, in the field of a classical source of colour ζ_s (a heavy quark) sees a potential

$$V(r) = \pm\left(|\zeta_s^+ \cdot \zeta_q|^2 - \frac{1}{N_c}\right) \left[\beta\frac{g^2}{8\pi r} - gf\sqrt{\frac{N_c}{8(N_c - 1)}} \times \ln\left(\exp(2mr) - 1 + \frac{m}{\mu}y_0^2\right)\right] , \tag{9}$$

with the upper sign holding for quark–quark interactions and the lower sign applying to quark–anti-quark interactions. The colour factor $\frac{1}{2}(|\zeta_s^+ \cdot \zeta_q|^2 - \frac{1}{N_c})$ defines a double cone around the direction of ζ_s. This double cone has an angle $\tan\theta_c = \sqrt{N_c - 1}$ against the symmetry axis specified by ζ_s, and separates domains of attraction from domains of repulsion: Quarks of colour ζ_q in the double cone are repelled and anti-quarks are attracted, while quarks with colour outside the cone are attracted and anti-quarks are repelled.

3. Potentials with an $1/r$ singularity at short distances and linear behaviour at large distances have been very successfully applied in investigations of the quarkonium spectrum, see e.g. [3] for two of the classical references in the field. Phenomenological tension parameters in heavy-light meson systems are of order $\sigma \simeq (430\,\text{MeV})^2$ [5], and in the present model the tension would be determined by the mass and coupling scale of the scalar field according to $\sigma \simeq gmf$.

Of course, the derivation of (8) does not imply that this yields the correct interquark potential in hadrons. What it does indicate is that direct couplings of scalar fields to Yang-Mills terms provide an interesting new paradigm for the description of confinement in gauge theories. This might be realized in QCD through a fundamental scalar, or eventually through a low energy effective scalar degree of freedom. The possibility of a fundamental scalar cannot be excluded, since the coupling scale f or the mass might be large enough to make such a scalar invisible to present day experiments. On the other hand, there exist scalar resonances in the hadronic spectrum whose rôle has not been understood yet.

On the level of a low energy effective scalar degree of freedom, one might speculate that the Lagrangian (1) with $\beta = 0$ is realized in the low energy regime through a QCD dilaton coupling to the trace anomaly, while at high energies the standard QCD Lagrangian would apply. Such a picture can be motivated, if one combines the old idea of scalar meson dominance of the trace of the energy-momentum tensor [6] with the QCD trace anomaly, as in [7]. Since the trace anomaly is proportional to the Yang-Mills term this could also justify the scalar gluon coupling in (1) with $\beta = 0$, and (9) then tells us how the dilaton changes the quark interaction potential. A disadvantage with this picture concerns the disappearance of the Coulomb term in the low energy regime.

The derivation of (9) from (1) indicates that direct couplings of scalars to Yang-Mills terms provide a new paradigm for confinement in gauge theories, and it seems well justified to dedicate more efforts to this approach to the confinement problem.

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